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MBF-003-1012008 Seat No. ________

B. Sc. (Sem. II) (CBCS) (W.I.F.-2016) Examination

March / April - 2018

Mathematics : Paper - MATH - 02 (A)
(Geometry, Calculus & Matrix Algebra)

(New Course)

Faculty Code : 003
Subject Code : 1012008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are **compulsory**.

1 (a) Answer the following : **4**

(1) Define Cylinder.

(2) Define Sphere.

(3) Write parametric equation of a Sphere.

(4) Define Great circle.

(b) Answer any **one** out of **two** : **2**

(1) Obtain Cartesian equation of sphere whose center is (–1, –2, 3) and radius 7.

(2) Find the equation of normal to the sphere $x^2 + y^2 + z^2 = 9$ at point A (1, 1, 1).

(c) Answer any **one** out of **two** : **3**

(1) Obtain the center and radius of the sphere

$\overline{r} - 2\overline{r}\left(-3, -3, 3\right) - 11 = 0.$

(2) Find the equation of cylinder parallel to Z-axis whose guiding curve is $x^2 + y^2 + z^2 = 9$

$x + 3y + 3z = 9.$

(d) Answer any **one** out of **two** : **5**

(1) Prove that the sphere

$x^2 + y^2 + z^2 + 4x + 2y - 59 = 0$ and

$x^2 + y^2 + z^2 - 8x + 6y - 6z + 9 = 0$ intersect each other and the intersection circle lies on the plane $6x - 2y + 3z + 34 = 0$.

(2) Find the equation of right circular cylinder having its base circle $x^2 + y^2 + z^2 = 9,\ x - y + z = 3$.

2 (a) Answer the following : **4**

(1) $\lim_{(x, y)\to(1, 1)} \frac{x^2 - y^2}{x - y} =$ __________.

(2) If $u = \frac{x^3 + y^3}{x^2 + y^2}$ then

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2\,xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \_\_\_\_\_\_\_\_\_\_.$$

(3) Define Continuous function of two variables.

(4) State Young's theorem for partial differentiation.

(b) Answer any one out of two : **2**

(1) Show that $\lim_{(x, y)\to(0, 0)} \frac{x^2 + y^2}{xy}$ does not exist.

(2) If $f(x, y) = \log\left(x^2 + y^2\right),\ x \neq 0,\ y \neq 0$ then find $f_x(1, 1)$ and $f_y(1, 1)$.

(c) Answer any **one** out of **two** : **3**

(1) If $f(x, y) = 0$ then obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(2) Discuss the continuity of a function $f(x, y)$ at (0, 0) where

$$f(x, y) = \frac{\sin\left(x^2 + y^2\right)}{x^2 + y^2} \quad ; \quad (x, y) \neq (0, 0)$$
$$= 0 \quad ; \quad (x, y) = (0, 0)$$

(d) Answer any **one** out of **two** : **5**

(1) State and prove Euler's theorem for homogeneous function of two variables.

(2) If $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$ then prove that :

(i) $$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(ii) $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

3 (a) Answer the following : **4**

(1) If $x = r\cos\theta$ and $y = r\sin\theta$ then $\frac{\partial(x, y)}{\partial(r, \theta)} =$ ______.

(2) Find critical point of function $f(x, y) = x^2 + 2y^2 - x$.

(3) Define Jacobean.

(4) If $J = \frac{\partial(x, y)}{\partial(u, v)}$ and $J' = \frac{\partial(u, v)}{\partial(x, y)}$ then $JJ' =$ _________.

(b) Answer any **one** out of **two** : **2**

(1) Find maxima and minima of function

$$f(x, y) = x^3 + y^3 - 3xy.$$

(2) If $f(x, y) = x^2 y - 3y$ then find the approximate value $f(5.12, 6.85)$.

(c) Answer any **one** out of **two** : **3**

(1) Divide 120 into three parts such that the sum of product of two numbers is maximum.

(2) Expand $e^x \cos y$ in power of x and y up to three degree.

(d) Answer any **one** out of **two** : **5**

(1) Find the greatest and smallest value of the function $f(x, y) = xy$ takes on ellipse

$$\frac{x^2}{8} + \frac{y^2}{2} = 1.$$

(2) If $u = x + y + x, uv = y + z, uvw = z$ then find

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} \text{ and } \frac{\partial(x, y, z)}{\partial(u, v, w)}.$$

4 (a) Answer the following : **4**

(1) The rank of every non singular matrix of order 'r' is ________.

(2) A square matrix A is said to be idempotent if ________.

(3) Define Hermitian Matrix.

(4) Define Singular Matrix.

(b) Answer any **one** out of **two** : **2**

(1) Prove that $A=\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is a nilpotent matrix of index 2.

(2) Prove that $A=\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is an orthogonal matrix.

(c) Answer any **one** out of **two** : **3**

(1) If $A=\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ find adj(A).

(2) Find inverse of $A=\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ using preliminary row operation.

(d) Answer any **one** out of **two** : **5**

(1) Prove that every square matrix can be uniquely express as sum of a symmetric and skew symmetric matrices.

(2) Find the rank of matrix $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ by reducing to normal form.

5 (a) Answer the following : **4**

(1) If λ is an eigen value of matrix A then what is the eigen value of A^{-1} ?

(2) Define Characteristic Polynomial.

(3) The equation $AX = B$ has unique solution if A is ___________.

(4) The equation $AX = 0$ has non zero solution if A is ___________.

(b) Answer any **one** out of **two** : **2**

(1) Find Eigen value of matrix $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$.

(2) Prove that 0 is a characteristic root of a matrix iff matrix is singular.

(c) Answer any **one** out of **two** : **3**

(1) Prove that Eigen values of Hermitian Matrix are real Numbers.

(2) Verify Cayley Hamilton theorem for matrix

$$A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}.$$

(d) Answer any **one** out of **two** : **5**

(1) State and prove Cayley-Hamilton theorem.

(2) Find the Eigen value and Eigen vectors of the

matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.